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Efficient TDOA/FDOA Based Moving Target Localization for Multi-Static Passive Radar

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Outline

- Background
  - Source Localization using a distributed sensor network
  - Related work
  - Main contributions
- Problem formulation
- Proposed approaches
  - Maximum likelihood estimator (MLE) using direct signal measurements
  - Constrained Cramer-Rao Bound (CRB)
  - Iterative re-weighted least square (IRLS) method using TDOA/FDOA estimates
- Numerical results
- Conclusions

Background

- Source localization is a fundamental signal processing problem encountered in a wide range of applications
- A standard localization approach is to employ a set of spatially distributed sensors to measure source signal
- Localization techniques
  - Non-coherent techniques - Based on Received Signal Strength (RSS)
  - Coherent techniques - Based on a suitable coherent model
    - Direct methods
      - Estimate the location/velocity parameters directly from sensor observations, e.g., maximum likelihood estimator (MLE)
    - Two-step methods
      - First estimate some intermediate parameters, e.g., time of arrival (TOA), time difference of arrival (TDOA), frequency difference of arrival (FDOA)
      - Then, solve the source localization problem by using intermediate parameters estimates

Related Work

- A direct method for location-only estimation (Weiss 2011)
  - Assumes source signal is stochastic with known statistics
- Two-step TDOA based methods for stationary target localization
  - TDOA is non-linearly dependent on the target location
  - One popular approach is to linearize the problem by using Taylor series expansion (Foy 1976) or adding redundant parameters (Chan & Ho 1994), (Ho 2012)
  - Another approach is to direct solve the non-linear estimation problem at higher complexity (Beck, Stoica & Li 2008)

- A. J. Weiss, "Direct geolocation of wideband emitters based on delay and Doppler," *IEEE TSP*, vol. 59, no. 6, pp. 2513–2521, June 2011.
- W. H. Foy, "Position-location solutions by Taylor-series estimation," *IEEE TAES*, no. 2, pp. 187–194, 1976.
- Y. T. Chan and K. C. Ho, "A simple and efficient estimator for hyperbolic location," *IEEE TSP*, vol. 42, no. 8, pp. 1905–1915, Aug 1994
- K. C. Ho, "Bias reduction for an explicit solution of source localization using TDOA," *IEEE TSP*, vol. 60, no. 5, pp. 2101–2114, May 2012
- A. Beck, P. Stoica, and J. Li, "Exact and approximate solutions of source localization problems," *IEEE TSP,* vol. 56, no. 5, pp. 1770–1778, May 2008

Related Work (cont.)

- Two-step TDOA/FDOA based methods for moving target localization
  - Since TDOA and FDOA are non-linearly dependent on target location and velocity, linearization techniques are popular, e.g., based on redundant parameters (Ho & Xu 2004)
  - Semi-definite relaxation based methods by reformulating the target localization problem into a convex semi-definite programming (Wang, Li & Ansari 2013)
  - Another method uses non-linear weighted least squares in two steps followed by bias reduction in each step (Wang et al. 2016)

- K. Ho and W. Xu, "An accurate algebraic solution for moving source location using TDOA and FDOA measurements," *IEEE TSP*, vol. 52, no. 9, pp. 2453–2463, 2004
- G. Wang, Y. Li, and N. Ansari, "A semidefinite relaxation method for source localization using TDOA and FDOA measurements," *IEEE TVT*, vol. 62, no. 2, pp. 853–862, Feb 2013
- G. Wang, S. Cai, Y. Li, and N. Ansari, "A bias-reduced nonlinear WLS method for TDOA/FDOA-based source localization," *IEEE TVT,* vol. 65, no. 10, pp. 8603–8615, Oct 2016

Main Contributions

- Developed a maximum likelihood estimator (MLE) to jointly estimate the location and velocity of a moving target
  - MLE is a direct method that employs signal measurements
  - Models the unknown source waveform as a deterministic process
  - Optimum, but computationally involved
- Developed an efficient two-step TDOA/FDOA based estimator for moving target localization
  - TDOA/FDOA estimates are obtained by 2D-FFT followed by local gradient search
  - An iterative re-weighted least square (IRLS) algorithm with a varying weighting matrix is used to determine the source location and velocity from TDOA and FDOA estimates
  - Does not require the TDOA/FDOA covariance matrix, which is required by most existing methods. The covariance matrix is usually unknown in practice

Problem Formulation

- Consider a distributed passive radar sensor network, where M widely separated sensors are utilized to receive signal emitted by a source
- Source is located at $\boldsymbol{u} = [x, y]^T$ and moving with a velocity $\boldsymbol{v} = [v_x, v_y]^T$
- Coordinates of the m-th sensor are $(x_m, y_m), m = 1,2, \dots, M$
- The **received signal** $r_m(t)$ at the m-th sensor can be written as

$$r_m(t) = \alpha'_m s'(t - \tau_m) e^{j2\pi f_m t} + w_m(t)$$

  - $s'(t)$: unknown source signal
  - α'_m: unknown channel coefficient
  - $w_m(t)$: additive zero-mean Gaussian noise
  - τ_m: delay from the source to the m-th sensor

$$\tau_m = d_m / c, d_m = \sqrt{(x - x_m)^2 + (y - y_m)^2}$$

  - f_m: Doppler frequency observed at the m-th sensor

$$f_m = -\frac{1}{\lambda} \frac{v_x(x - x_m) + v_y(y - y_m)}{d_m}$$

Reparameterization

- Choose sensor with largest observation energy level as a **reference sensor**
- Delays τ_m and Doppler frequencies f_m can be reparametrized as **TDOAs** $\tau_{m1} = \tau_m - \tau_1$ and **FDOAs** $f_{m1} = f_m - f_1$
- The received signal is

$$r_m(t) = \alpha_m s(t - \tau_{m1}) e^{j2\pi f_{m1} t} + w_m(t)$$
$$\alpha_m = \alpha'_m e^{-j2\pi f_1 \tau_{m1}}, s(t) = s'(t - \tau_1) e^{j2\pi f_1 t}$$

- After sampling, the **discrete-time received signal**

$$\boldsymbol{r}_m = \alpha_m \boldsymbol{\Phi}(\tau_{m1}, f_{m1}) \boldsymbol{s} + \boldsymbol{w}_m$$

$$\boldsymbol{\Phi}(\tau_{m1}, f_{m1}) = \boldsymbol{W}(f_{m1} T_s) \boldsymbol{T}^H \boldsymbol{W}(-\tau_{m1} \Delta f) \boldsymbol{T}$$
$$[\boldsymbol{W}(a)]_{p,p} = e^{j2\pi(p-1)a}, \quad p = 1, 2, \ldots, N$$

$\boldsymbol{T}$ is an $N \times N$ DFT matrix

Maximum Likelihood Estimator

- Likelihood function of received signals

$$c_1(\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{\alpha}, \boldsymbol{s}, \sigma^2) = -MN \ln \sigma^2 - \frac{1}{\sigma^2} \sum_{i=1}^{M} \|\boldsymbol{r}_m - \alpha_m \boldsymbol{\Phi}_{m1} \boldsymbol{s}\|^2$$

- After substituting the estimates of α_m, σ^2, and $\boldsymbol{s}$, we obtain

$$c_2(\boldsymbol{u}, \boldsymbol{v}) = -MN \ln \left(\sum_{m=1}^{M} \|\boldsymbol{r}_m\|^2 - \lambda_{\max} \right)$$

where $\lambda_{\max}$ is the largest eigenvalue of matrix

$$\boldsymbol{Z}^H \boldsymbol{Z} = \begin{bmatrix} \|\boldsymbol{r}_1\|^2 & \cdots & \boldsymbol{r}_1^H \boldsymbol{\Phi}_{M1}^H \boldsymbol{r}_M \\ \vdots & \ddots & \vdots \\ \boldsymbol{r}_M^H \boldsymbol{\Phi}_{M1} \boldsymbol{r}_1 & \cdots & \|\boldsymbol{r}_M\|^2 \end{bmatrix}$$

- MLE estimates the source location and velocity by

$$\{\hat{\boldsymbol{u}}, \hat{\boldsymbol{v}}\} = \arg\max_{\boldsymbol{u},\boldsymbol{v}} \lambda_{\max}\left(\boldsymbol{Z}^H \boldsymbol{Z}\right)$$

which requires a 4-dimensional (4-D) search over the parameter space

Constrained CRB

- Due to an inherent multiplicative ambiguity between the source waveform and the channel gain, a constrained CRB is derived by imposing a constraint on the source waveform
- The Fisher information matrix (FIM) $J(\theta)$ for the unknown parameter

$$\boldsymbol{\theta} = [\boldsymbol{u}^T, \boldsymbol{v}^T, \boldsymbol{\alpha}_R, \boldsymbol{\alpha}_I, \boldsymbol{s}_R^T, \boldsymbol{s}_I^T, \sigma^2]^T$$

is expressed by FIM $J(\vartheta)$ of the new parameter ϑ

$$\boldsymbol{\vartheta} = [\boldsymbol{\tau}_u, \boldsymbol{f}_v, \boldsymbol{\alpha}_R, \boldsymbol{\alpha}_I, \boldsymbol{s}_R^T, \boldsymbol{s}_I^T, \sigma^2]^T$$

according to the chain rule: $\boldsymbol{J}(\boldsymbol{\theta}) = (\nabla_{\boldsymbol{\theta}}\boldsymbol{\vartheta})\boldsymbol{J}(\boldsymbol{\vartheta})(\nabla_{\boldsymbol{\theta}}\boldsymbol{\vartheta})^T$

- Constrained CRB: $\mathrm{CRB}[\theta | h(\boldsymbol{\theta}) = 0] = \boldsymbol{Q}[\boldsymbol{Q}^T \boldsymbol{J}(\theta)\boldsymbol{Q}]^{-1}\boldsymbol{Q}^T$

with constraint $h(\boldsymbol{\theta}) = s_R[1] + j s_I[1] - (a_1 + j a_2) = 0$ and $\dfrac{\partial h(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}^T}\boldsymbol{Q} = 0$

IRLS: TDOA and FDOA Estimation

- Maximum likelihood estimates of the TDOA and FDOA $\{\tau_{m1}, f_{m1}\}$ for the m-th pair of observations $\{\boldsymbol{r}_1, \boldsymbol{r}_m\}$ are given by

$$\left\{\widehat{\tau}_{m1}, \widehat{f}_{m1}\right\} = \arg \max_{\tau_{m1}, f_{m1}} \left| \boldsymbol{r}_1^H \boldsymbol{\Phi}_{m1}^H \boldsymbol{r}_m \right|$$

- The cost function can be expressed as

$$\boldsymbol{r}_1^H \boldsymbol{\Phi}_{m1}^H \boldsymbol{r}_m = \frac{1}{N} \sum_{p=1}^{N} \sum_{q=1}^{N} [\boldsymbol{P}]_{p,q} e^{-j2\pi(q-1)f_{m1}T_s} e^{j2\pi(p-1)\tau_{m1}\Delta f}$$

$$\boldsymbol{P} = N\Theta_1 \boldsymbol{T} \Theta_m, \ [\Theta_1]_{n,n} = [(\boldsymbol{T}\boldsymbol{r}_1)^*]_n, \text{ and } [\Theta_m]_{n,n} = [\boldsymbol{r}_m]_n$$

- TDOA and FDOA estimates using a 2-D FFT and local gradient based search

$$\widehat{\tau}_{m1} = \frac{M_\tau - p + 1}{\Delta f M_\tau}, \quad \widehat{f}_{m1} = \frac{q-1}{T_s M_f}$$

IRLS: Source Location Estimation

- Define range difference vector $\hat{d} = [\hat{d}_{21}, \dots, \hat{d}_{M1}]$ and

$$d_{m1} = \sqrt{(x - x_m)^2 + (y - y_m)^2} - \sqrt{(x - x_1)^2 + (y - y_1)^2} = c\tau_{m1}$$

- Non-linear function of source location parameter $\boldsymbol{u}$

$$\hat{\boldsymbol{d}} = \boldsymbol{g}(\boldsymbol{u}) + \boldsymbol{e}_u$$

where $\boldsymbol{g}(\boldsymbol{u}) = [g_2(\boldsymbol{u}), \dots, g_M(\boldsymbol{u})]^T$ and $g_m(\boldsymbol{u}) = d_{\mathrm{m1}}$

- First-order Taylor expansion of $\boldsymbol{g}(\boldsymbol{u})$ at $\boldsymbol{u}^{(l-1)}$ yields

$$\hat{\boldsymbol{d}} \approx \boldsymbol{g}(\boldsymbol{u}^{(l-1)}) + \boldsymbol{G}_p^{(l-1)} \Delta\boldsymbol{u}^{(l)} + \boldsymbol{e}_u$$

$$\Delta\boldsymbol{u}^{(l)} = \begin{bmatrix} \Delta x^{(l)} \\ \Delta y^{(l)} \end{bmatrix}, \quad \boldsymbol{G}_p^{(l-1)} = \begin{bmatrix} \frac{\partial g_2(\boldsymbol{u}^{(l-1)})}{\partial x} & \frac{\partial g_2(\boldsymbol{u}^{(l-1)})}{\partial y} \\ \vdots & \vdots \\ \frac{\partial g_{(M)}(\boldsymbol{u}^{(l-1)})}{\partial x} & \frac{\partial g_{(M)}(\boldsymbol{u}^{(l-1)})}{\partial y} \end{bmatrix}$$

IRLS: Source Location Estimation

- $\Delta \boldsymbol{u}^{(l)}$ can be obtained by a weighted least squares fitting

$$\Delta \boldsymbol{u}^{(l)} = \left(\boldsymbol{G}_p^{(l-1)H} \boldsymbol{R}_u^{(l-1)} \boldsymbol{G}_p^{(l-1)} \right)^{-1} \boldsymbol{G}_p^{(l-1)H} \boldsymbol{R}_u^{(l-1)} \left(\hat{\boldsymbol{d}} - \boldsymbol{g}(\boldsymbol{u}^{(l-1)}) \right)$$

  with a weighting matrix $\boldsymbol{R}_u^{(l-1)}$

- A varying diagonal weighting matrix is employed to apply larger weight to the sensor with better measurement

$$\boldsymbol{R}_u^{(l-1)}(m-1, m-1) = \left[(\hat{d}_{m1} - g_m(\boldsymbol{u}^{(l-1)})) \right]^{-2}, \quad m = 2, \ldots, M$$

- The source location estimate can be updated by

$$\boldsymbol{u}^{(l)} = \boldsymbol{u}^{(l-1)} + \Delta \boldsymbol{u}^{(l)}$$

- The iterative process ends when $\Delta \boldsymbol{u}^{(l)} < \varepsilon$

IRLS: Source Velocity Estimation

- A similar **iterative reweighted procedure is implemented for** source velocity estimation after the location estimate is obtained
- Define **range difference** $\widehat{\dot{\boldsymbol{d}}} = \left[\hat{\dot{d}}_{21}, \dots, \hat{\dot{d}}_{M1}\right]^T$, where

$$\dot{d}_{m1} = \frac{v_x(x - x_m) + v_y(y - y_m)}{d_m} - \frac{v_x(x - x_1) + v_y(y - y_1)}{d_1} = -\lambda f_{m1}$$

- Then, we have

$$\widehat{\dot{\boldsymbol{d}}} = \boldsymbol{H_u v} + \boldsymbol{e_v}$$

$$\boldsymbol{H_u} = \begin{bmatrix} \frac{x-x_2}{d_2} - \frac{x-x_1}{d_1} & \frac{y-y_2}{d_2} - \frac{y-y_1}{d_1} \\ \frac{x-x_3}{d_3} - \frac{x-x_1}{d_1} & \frac{y-y_3}{d_3} - \frac{y-y_1}{d_1} \\ \vdots & \vdots \\ \frac{x-x_M}{d_M} - \frac{x-x_1}{d_1} & \frac{y-y_M}{d_M} - \frac{y-y_1}{d_1} \end{bmatrix}$$

- Notice that that $\widehat{\dot{\boldsymbol{d}}}$ is linearly related to $\boldsymbol{v}$

IRLS: Source Velocity Estimation

- Substituting the source location estimate $\hat{\boldsymbol{u}}$ into the equation and letting $\boldsymbol{v}^{(l-1)}$ denote the results from the $(l-1)$-st iteration, we obtain

$$\boldsymbol{v}^{(l)} = \left[\widehat{\boldsymbol{H}}_{\boldsymbol{u}}^{H} \boldsymbol{R}_{v}^{(l-1)} \widehat{\boldsymbol{H}}_{\boldsymbol{u}}\right]^{-1} \widehat{\boldsymbol{H}}_{\boldsymbol{u}}^{H} \boldsymbol{R}_{v}^{(l-1)} \hat{\dot{\boldsymbol{d}}}$$

with a similar varying diagonal weighting matrix which apply larger weight to the sensor with better measurement

$$\boldsymbol{R}_{v}^{(l-1)}(m-1, m-1) = \left[(\hat{\dot{d}}_{m1} - \widehat{\boldsymbol{H}}_{\boldsymbol{u}}(m,:)\boldsymbol{v}^{(l-1)}\right]^{-2}, \quad m = 2, \ldots, M$$

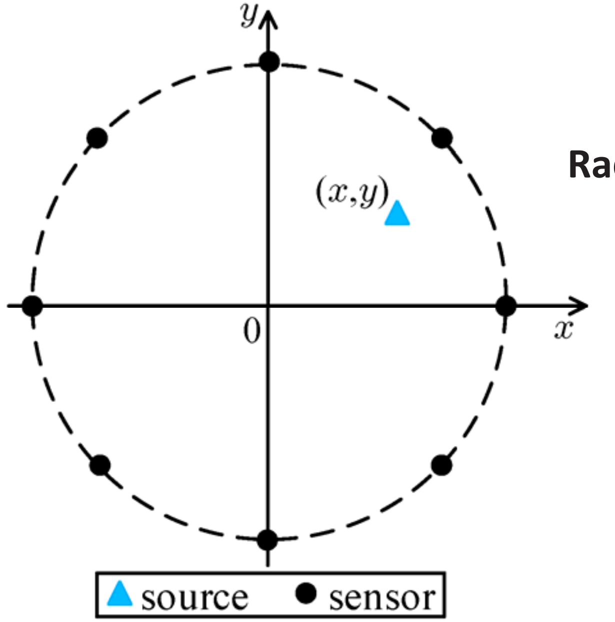
- The iterative process ends when $\boldsymbol{v}^{(l)} - \boldsymbol{v}^{(l-1)} < \varepsilon$

Simulation Setup

- $M = 8$ sensors uniformly spaced on a circle
- The signal-to-noise ratio (SNR) for the m-th sensor

$$\text{SNR}_m = \frac{\alpha_m^2}{\sigma^2} = \frac{d_1^2}{d_m^2}\text{SNR}_1, \quad m = 2, \ldots, M$$

$$\text{SNR}_1 = \frac{\alpha_1^2}{\sigma^2}$$



Radius = $33.33cT_s$

- Source localization methods:
  - MLE with two different initializations
    - MLE (init-1) - Initialized by a practical method
    - MLE (init-2) - Initialized by true source parameters
  - IRLS: Proposed method
  - NWLS-BiasSub (Wang et al. 2016) - A non-linear weighted least square (NWLS) method with bias subtraction

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Simulation Setup

- Two cases of SNR spread are considered by changing relative location between source and sensors

$$\text{SNR}_m = \frac{\alpha_m^2}{\sigma^2} = \frac{d_1^2}{d_m^2}\text{SNR}_1$$

  - Case 1 (small SNR spread):
    - $\boldsymbol{u} = [4cT_s, 6cT_s]^T$, close to the center of the circle
    - small SNR spread for all sensors
  - Case 2 (large SNR spread):
    - $\boldsymbol{u} = [19.3cT_s, 23.3cT_s]^T$, close to the reference sensor
    - larger SNR spread among all sensors

- Initialization
  - MLE (init-1) and IRLS: intersecting the hyperbolas associated with two best TDOA estimates
  - MLE (init-2): true source parameters
  - NWLS-BiasSub: using the FIM along with initial parameter estimates as TDOA/FDOA covariance matrix

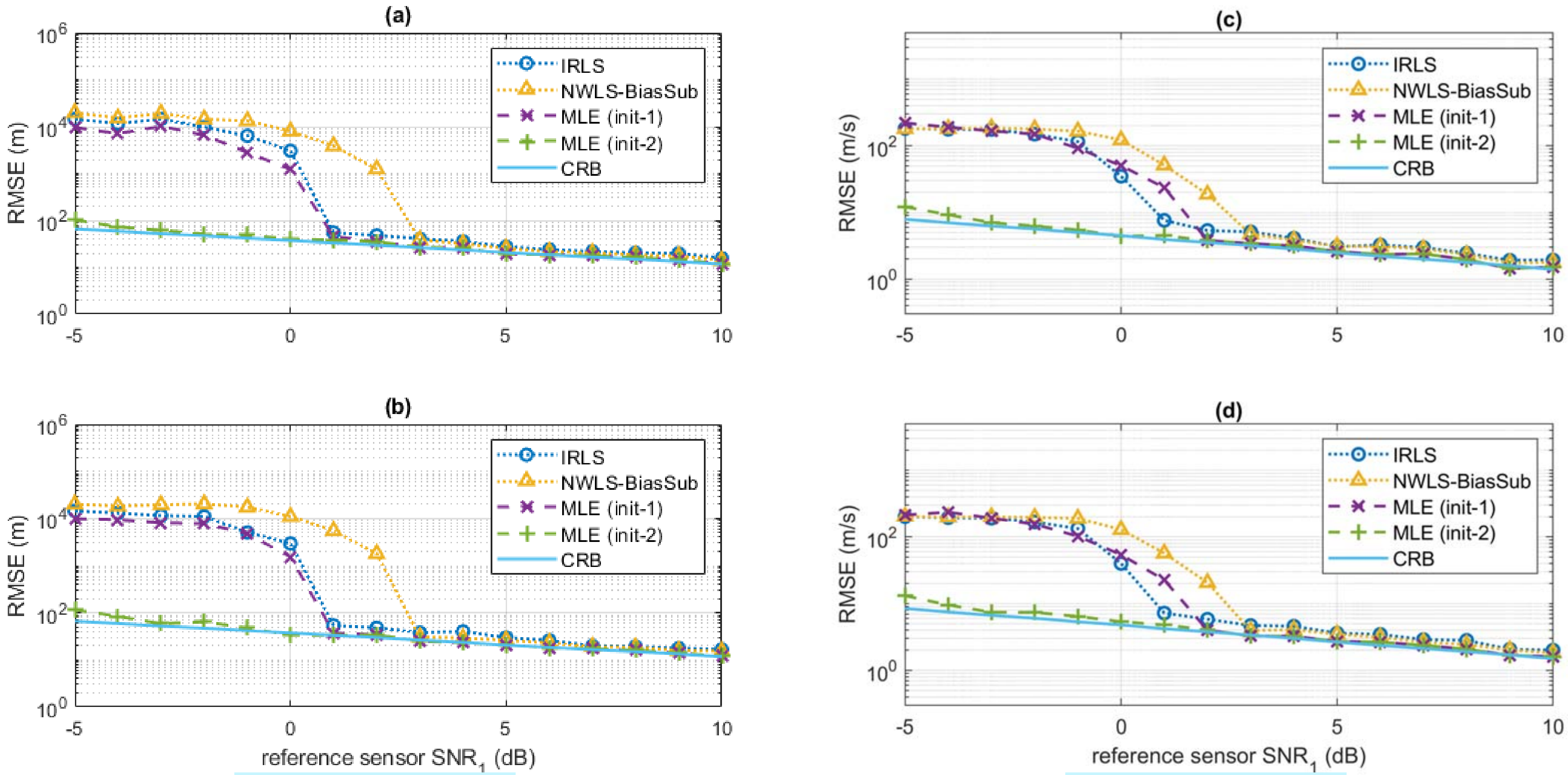
Numerical Results – Small SNR Spread

- **Case 1**: Source is located relatively **close to the center** of the circle

$N = 100$

x Component

y Component



Location

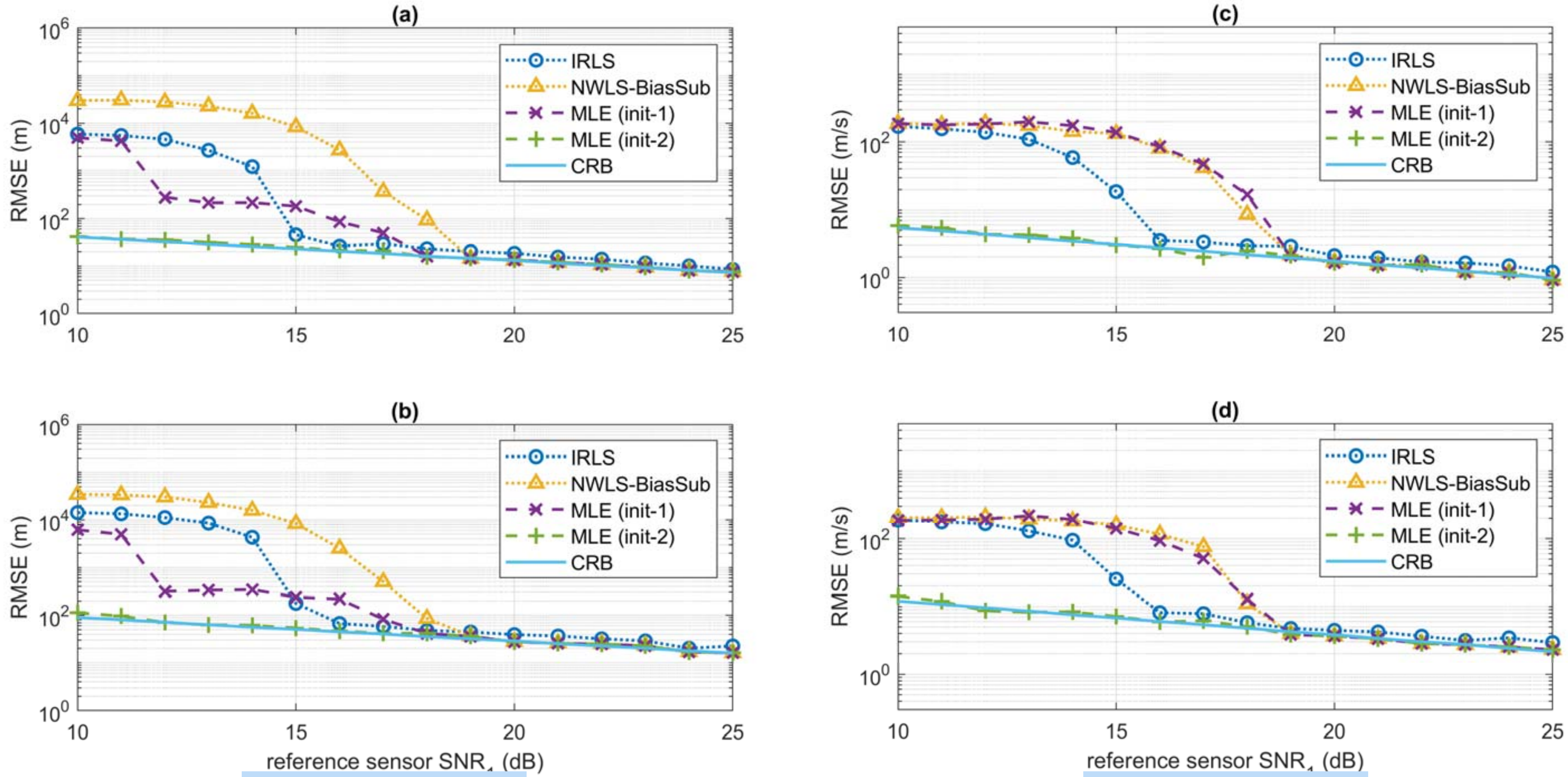
Velocity

Numerical Results – Large SNR Spread

- **Case 2**: Source is located **close to one particular sensor** (reference sensor) $N = 100$

x Component

y Component

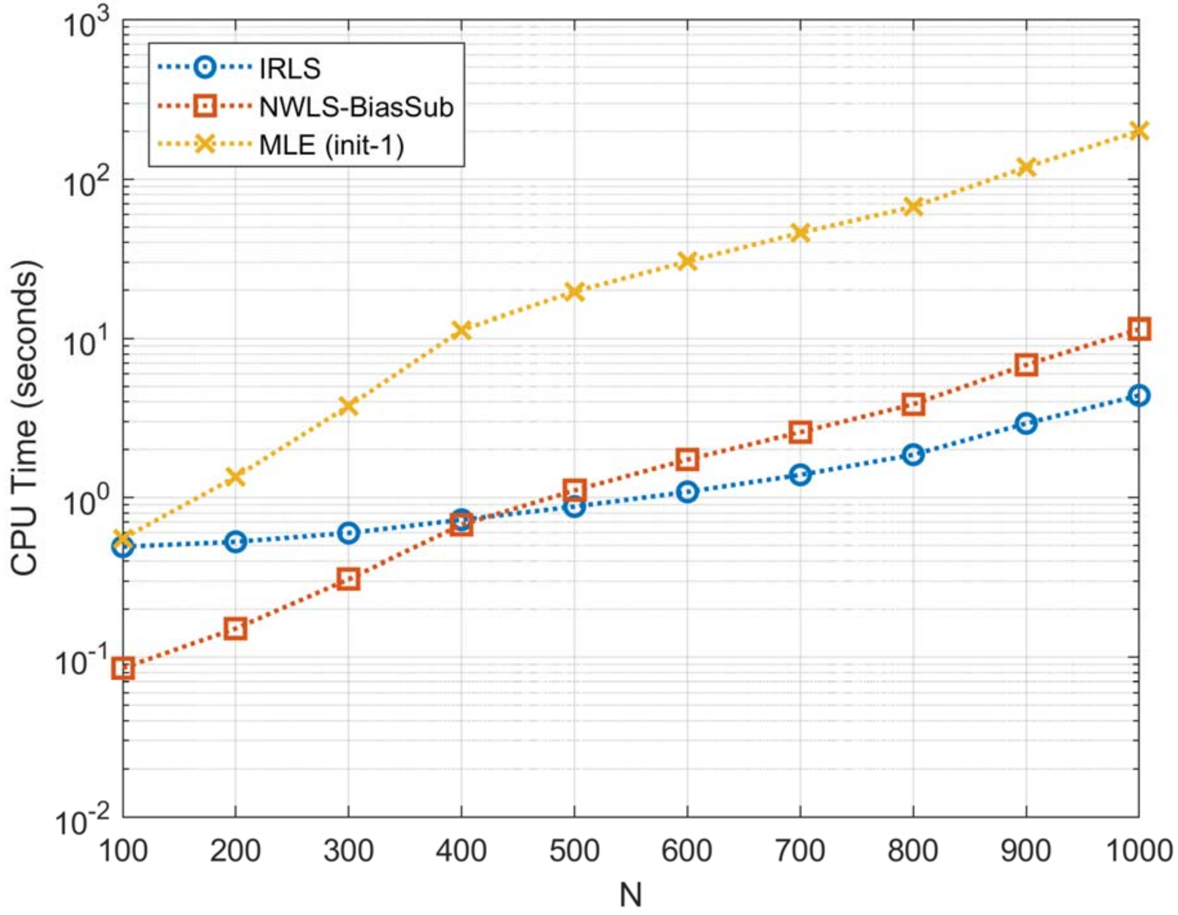


Location

Velocity

Numerical Results

- **Computational complexity in Case 2 when SNR$_1$ $= 20$ dB**



Concluding Remarks

- Developed a signal based MLE for moving target localization by assuming the source waveform is deterministic unknown
- Derived a constrained CRB associated with the source localization problem was provided
- Proposed a computationally efficient TDOA/FDOA based IRLS method that employs 2D-FFT, interpolation, and iterative reweighting to solve the localization problem
- Numerical results show that IRLS has a lower SNR threshold and compares favorably with several well-known TDOA/FDOA based solutions

QUESTIONS?